



马来西亚留台成功大学校友会

主 办

2013 年

第三十届全国华文独中数理学识比赛

考生指示：

- (一) 解答所有问题。
- (二) 将正确答案在答案纸上的圆圈内「塗黑」，每题只准给一个答案。
- (三) 正确的答案得三分，错误的答案扣一分，不做答的零分。

**INSTRUCTIONS TO CANDIDATES**

1. Attempt all questions.
2. Pick the correct answer and make a mark “X” in the circle provided in the answer sheet. Only one answer is allowed for each question.
3. Three marks for a correct answer, one mark will be deducted for each wrong answer. No mark will given to each question not attempted.

1. 2013 個人排隊等候進入演唱會。在隊伍中，每兩為男士中間最少有三位女士。請問隊伍中男士最多有幾位？

2013 people stand in a queue at a concert entrance. There are at least three women between any two men. What is the largest possible number of men in the queue?

- (a) 504
- (b) 505
- (c) 617
- (d) 618
- (e) 以上皆非 None of the above

2. 以下哪一個敘述對三個連續自然數不見得正確？

Which of the following statements is not always true for three consecutive natural numbers?

- (a) 最少有一為偶數。At least one is even.
- (b) 恰好有一數可以被 3 整除。Exactly one of them is divisible by 3.
- (c) 有一數可被 6 整除。One of them is divisible by 6.
- (d) 三數的乘積可被 6 整除。The product is divisible by 6.
- (e) 以上皆非 None of the above

3. 將 9 個蘋果分給甲、乙和丙三人。如果要求甲最少得到 3 個，乙、丙每人各最少得到 2 個，而丙最多得到 3 個，請問有幾種不同分法？

We want to divide 9 apples among three persons A, B and C in such a way that A gets at least 3 oranges, B and C at least 2 each, and C at most 3. How many different ways can we do it?

- (a) 4
- (b) 5
- (c) 6
- (d) 7
- (e) 以上皆非 None of the above

4. 假設  $a$  和  $b$  是實數，且  $a + b = 5$  及  $ab = 2$ 。請問  $a^4 + b^4$  為何？

Let  $a$  and  $b$  be real numbers such that  $a + b = 5$  and  $ab = 2$ . What is  $a^4 + b^4$ ?

- (a) 433
- (b) 437
- (c) 609
- (d) 625
- (e) 以上皆非 None of the above

5. 方程式  $\sqrt{x^4 + 16} = x^2 - 4$  有多少實數解?

Consider the equation  $\sqrt{x^4 + 16} = x^2 - 4$ . How many (real) solutions does it has?

- (a) 0
- (b) 1
- (c) 2
- (d) 3
- (e) 以上皆非 None of the above

6. 甲車的每個輪胎周長為 200 公分, 乙車的每個輪胎周長為 250 公分。經過一段 20 公里的旅程後, 甲、乙二車的輪子總迴轉數差多少?

The tyres of car *A* have circumference 200 cm each. The tyres of car *B* has circumference 250 cm each. Both of them take a journey of 20 km. What is the difference in the number of revolutions made by a wheel of car *A* and a wheel of car *B*?

- (a) 800
- (b) 1000
- (c) 2000
- (d) 4000
- (e) 以上皆非 None of the above

7. 以 10 為對數底,  $1000^{\log 2 + \log 3}$  的值為何?

Consider base 10 for the logarithms. What is  $1000^{\log 2 + \log 3}$ ?

- (a) 6
- (b) 36
- (c) 216
- (d) 1296
- (e) 以上皆非 None of the above

8. 讓  $100!/(18)^n$  不是整數的最小的正整數  $n$  為何?

Let  $n$  be the smallest integer  $n$  such that  $100!/(18)^n$  is not an integer. What is  $n$ ?

- (a) 23
- (b) 24
- (c) 25
- (d) 26
- (e) 以上皆非 None of the above

9. 計算  $\cos(20^\circ) - \cos(40^\circ) + \cos(60^\circ) - \cos(80^\circ)$ .

Evaluate  $\cos(20^\circ) - \cos(40^\circ) + \cos(60^\circ) - \cos(80^\circ)$ .

(a)  $\sqrt{3}/2$

(b)  $\sqrt{3}/3$

(c)  $2/3$

(d)  $1/2$

(e) 以上皆非 None of the above

10. 令  $x$  和  $y$  為正整數滿足  $x^2 - y^2 = 196$ . 則  $x^2 + y^2$  的值可能為何?

Let  $x$  and  $y$  be positive integers such that  $x^2 - y^2 = 196$ . What could  $x^2 + y^2$  possible be?

(a)  $19208\frac{1}{2}$

(b) 4808

(c)  $1208\frac{1}{2}$

(d)  $416\frac{1}{2}$

(e) 以上皆非 None of the above

11. 數個正整數加總後為 10。其乘積的最大可能是多少?

There are several (not necessarily different) positive integers sum up to 10. What is the largest possible value of their product?

(a) 25

(b) 32

(c) 36

(d) 54

(e) 以上皆非 None of the above

12. 令  $S$  為所有正整數  $n$  滿足  $1 \leq n \leq 2013$  且 1 沒有出現在其位數內。則  $S$  有多少整數在其中?

Let  $S$  be the set of all integers  $n$  with  $1 \leq n \leq 2013$  such that 1 does not occur in the decimal expansion of  $n$ . How many integers are in  $S$ ?

(a) 1999

(b) 1005

(c) 737

(d) 528

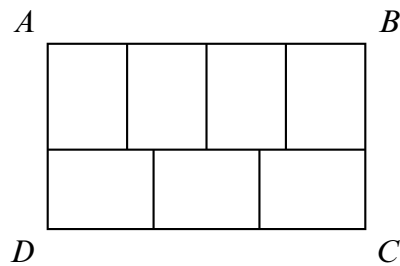
(e) 以上皆非 None of the above

13. 一個二次多項式  $p(x)$  滿足  $p(0) = 3$ ,  $p(1) = 5$ ,  $p(2) = 8$ 。則  $p(5)$  為何?

Let  $p(x)$  be a quadratic polynomial with  $p(0) = 3$ ,  $p(1) = 5$ ,  $p(2) = 8$ . Then  $p(5)$  is?

- (a) 21
  - (b) 22
  - (c) 23
  - (d) 24
  - (e) 以上皆非 None of the above
14. 面積為 336 的長方形  $ABCD$  可被分割為七個較小但相同大小的長方形, 如下圖所示。請問  $ABCD$  的邊長為何?

Suppose that the rectangle  $ABCD$  has area 336, and can be divided into seven smaller and equal rectangles, as shown in the figure. What is the perimeter of the rectangle  $ABCD$ ?



- (a) 76
  - (b) 78
  - (c) 82
  - (d) 90
  - (e) 以上皆非 None of the above
15. 假設  $f(x)$  是一個函數, 且對所有實數  $x$  滿足  $f(x) + 5f(1/x) = 3 + x$ 。請問  $f(4)$  為何?
- Let  $f(x)$  be a function satisfying  $f(x) + 5f(1/x) = 3 + x$  for all nonzero real numbers  $x$ . What is  $f(4)$ ?
- (a)  $5/4$
  - (b) 1
  - (c) 2
  - (d)  $37/96$
  - (e) 以上皆非 None of the above

16. 假如 7 個人能在 7 分鐘內吃掉七個比薩, 則 14 個人在 14 分鐘內可以吃掉幾個比薩?

Assume that 7 men can eat 7 pizzas in 7 minutes. How many pizzas could 14 men eat in 14 minutes?

- (a) 14
- (b) 21
- (c) 28
- (d) 35
- (e) 以上皆非 None of the above

17. 下列方程式有多少個實數解  $x$ ?

How many real numbers  $x$  are solutions of

$$x = \left(x - \frac{1}{x}\right)^{1/2} + \left(1 - \frac{1}{x}\right)^{1/2} ?$$

- (a) 0
- (b) 1
- (c) 2
- (d) 3
- (e) 以上皆非 None of the above

18. 一個袋子中有五個相同的球, 3 個是紅色的 2 個是綠色的。由袋中任意將球一一拿出來, 第五個拿出來的球是紅色的機率是多少?

There are 5 balls in a bag, 3 red and 2 green. The balls are taken out one by one randomly. What is the probability that the fifth ball is a red one?

- (a) 1/5
- (b) 1/3
- (c) 2/5
- (d) 3/5
- (e) 以上皆非 None of the above

19. 有多少實數序對  $(x, y, z)$  滿足  $xy = z$ ,  $xz = y$  及  $yz = x$ ?

How many triples of real numbers  $(x, y, z)$  are there such that  $xy = z$ ,  $xz = y$  and  $yz = x$ ?

- (a) 3
- (b) 5
- (c) 60
- (d) 無窮多 Infinitely many
- (e) 以上皆非 None of the above

20. 假設  $a, b, c$  為三個非零的數, 而多項式  $p(x) = x^3 - ax^2 + bx - c$  可分解為  $(x - a)(x - b)(x - c)$ 。請問  $p(2)$  為多少?

Let  $a, b, c$  be three nonzero numbers such that the polynomial  $p(x) = x^3 - ax^2 + bx - c$  factors as  $(x - a)(x - b)(x - c)$ . What is the value  $p(2)$ ?

- (a)  $-7$
- (b)  $0$
- (c)  $7$
- (d)  $9$
- (e) 以上皆非 None of the above

21. 甲、乙、丙同時做一測驗。甲得分 80, 乙得分比三人的平均高出 10 分, 丙得分比三人的平均低了 16 分。請問三人的平均為多少?

Alice, Bob, and Cathy took an exam. Alice scored 80 points. Bob scored 10 points more than the average score of the three, while Cathy scored 16 points less than the average of the three. What is the average score of the three?

- (a) 74
- (b)  $74\frac{2}{3}$
- (c)  $75\frac{1}{3}$
- (d) 76
- (e) 以上皆非 None of the above

22. 假設函數  $f$  滿足  $f(xy) = f(x)/y$ ,  $x$  and  $y$  為正實數。如果  $f(5) = 10$ , 則  $f(8)$  為何?

Let the function  $f$  satisfy  $f(xy) = f(x)/y$  for all positive numbers  $x$  and  $y$ . If  $f(5) = 10$ , then what is the value of  $f(8)$ ?

- (a) 4

- (b)  $32/5$
- (c)  $16/5$
- (d)  $25/4$
- (e) 以上皆非 None of the above

23. 下列關於  $f(x) = \frac{-4x - 5}{2x - 3}$  的敘述何者為真?

Which of the following statement about  $f(x) = \frac{-4x - 5}{2x - 3}$  is true?

- (a)  $f$  不是一對一。  $f$  is not one-to-one.
- (b)  $f$  是一對一, 且  $f^{-1}(x) = \frac{-3x + 5}{-2x - 4}$ 。  $f$  is one-to-one, and  $f^{-1}(x) = \frac{-3x + 5}{-2x - 4}$ .
- (c)  $f$  是一對一, 且  $f^{-1}(x) = \frac{-3x + 5}{-2x + 4}$ 。  $f$  is one-to-one, and  $f^{-1}(x) = \frac{-3x + 5}{-2x + 4}$ .
- (d)  $f$  是一對一, 且  $f^{-1}(x) = \frac{-3x + 5}{2x - 4}$ 。  $f$  is one-to-one, and  $f^{-1}(x) = \frac{-3x + 5}{2x - 4}$ .
- (e) 以上皆非 None of the above

24. 三角形  $ABC$  中的  $AC = 7$ .  $D$  在  $AB$  邊上使得  $AD = BD = CD = 5$ . 請問  $BC$  邊長多少?

In triangle  $ABC$ ,  $AC = 7$ .  $D$  lies on  $AB$  such that  $AD = BD = CD = 5$ . What is  $BC$ ?

- (a)  $\sqrt{32}$
- (b) 7
- (c)  $\sqrt{51}$
- (d)  $\sqrt{63}$
- (e) 以上皆非 None of the above

25. 一個數列滿足  $a_n = a_{n-1} + 3a_{n-2} + a_{n-3}$ , 且  $a_0 = a_1 = a_2 = 1$ . 則  $a_{2013}$  被 7 除了之後的餘數是多少?

Consider a sequence given by  $a_n = a_{n-1} + 3a_{n-2} + a_{n-3}$ , where  $a_0 = a_1 = a_2 = 1$ . What is the remainder of  $a_{2013}$  divided by 7?

- (a) 1
- (b) 2
- (c) 4
- (d) 5
- (e) 以上皆非 None of the above